研究開発交流会：令和2年6月18日

非等方性乱流モデルによる熱流動解析

杉山　均

宇都宮大学

工学部

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- 乱流場での支配方程式
- 境界適合座標系
- 粗面壁の乱流熱伝達現象解析
- Thrust vectoring nozzle 内の乱流解析
- 魚の泳法メカニズムと熱流動解析

乱流場での支配方程式

- 速度，圧力，温度を平均と変動に分解

$$U_i = \overline{U_i} + u_i \qquad P = \overline{P} + p \qquad T = \overline{T} + T'$$

- 運動量方程式(Navier-Stokes方程式)

$$\frac{\partial \overline{U_i}}{\partial t} + \overline{U_k}\frac{\partial \overline{U_i}}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_k}\left(\nu\frac{\partial \overline{U_i}}{\partial x_k} \boxed{-\overline{u_i u_j}}\right)$$

レイノルズ応力

- エネルギー方程式

$$\frac{\partial \overline{T}}{\partial t} + \overline{U_k}\frac{\partial \overline{T}}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\kappa\frac{\partial \overline{T}}{\partial x_k} \boxed{-\overline{u_i T'}}\right)$$

乱流熱流束

レイノルズ応力，乱流熱流束輸送方程式

- **レイノルズ応力輸送方程式**

圧力・歪相関項

$$\frac{\partial \overline{u_i u_j}}{\partial t} + U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} = -\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} - \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \boxed{\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}}$$

$$-\frac{\partial}{\partial x_k}\left[\overline{u_i u_j u_k} - \nu\frac{\partial \overline{u_i u_j}}{\partial x_k} - \overline{\frac{p}{\rho}\left(\delta_{jk}u_i+\delta_{ik}u_j\right)}\right] - 2\nu\overline{\frac{\partial u_i \partial u_j}{\partial x_k \partial x_k}}$$

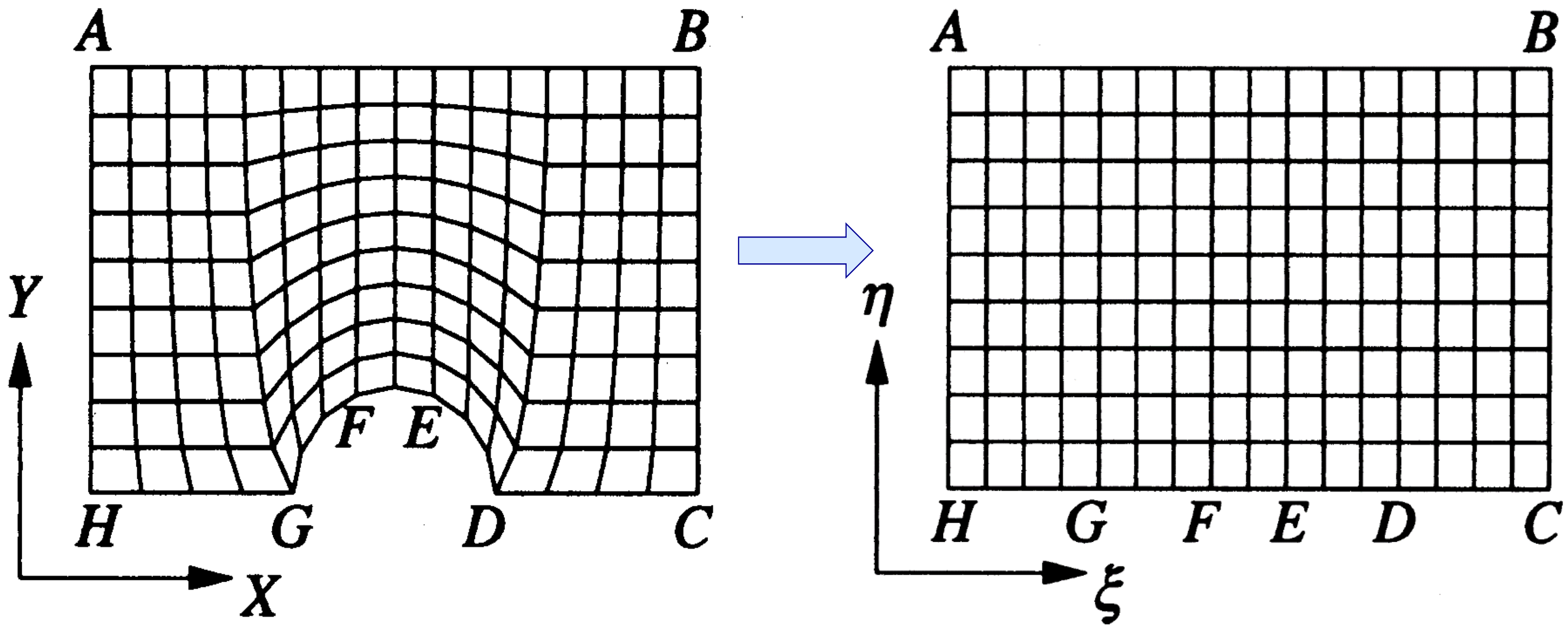
- **乱流熱流束輸送方程式**

圧力・温度勾配相関項

$$\frac{\partial \overline{u_i T'}}{\partial t} + U_k\frac{\partial \overline{u_i T'}}{\partial x_k} = -\overline{u_i u_j}\frac{\partial T}{\partial x_j} - \overline{u_j T'}\frac{\partial U_i}{\partial x_j} + \boxed{\overline{\frac{p}{\rho}\left(\frac{\partial T'}{\partial x_i}\right)}}$$

$$-\frac{\partial}{\partial x_j}\left[\overline{u_i u_j T'} + \overline{\frac{p}{\rho}T'\delta_{ij}}\right] - (\nu+\alpha)\overline{\frac{\partial T'\partial u_i}{\partial x_j \partial x_j}}$$

境界適合座標系



物理空間　　　計算空間

座標変換

- 変換定理

$$\frac{\partial}{\partial x_i}=\frac{\partial\xi}{\partial x_i}\frac{\partial}{\partial\xi}+\frac{\partial\eta}{\partial x_i}\frac{\partial}{\partial\eta}+\frac{\partial\zeta}{\partial x_i}\frac{\partial}{\partial\zeta}$$

- 運動量方程式(Navier-Stokes方程式)

$$\frac{\partial U_i}{\partial t}+\frac{\partial U_iU_j}{\partial x_j}=-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\nu\frac{\partial U_i}{\partial x_j}-\overline{u_iu_j}\right)$$

$$-\overline{u_iu_j}=\left(\frac{b}{a}\right)_{ij}\frac{\partial U_i}{\partial x_j}-\left(\frac{c}{a}\right)_{ij}\qquad d_{ij}=\nu+\left(\frac{b}{a}\right)_{ij},\quad e_{ij}=\left(\frac{c}{a}\right)_{ij}$$

$$\frac{\partial U_i}{\partial t}+\frac{\partial U_iU_j}{\partial x_j}=-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}\left(d_{ij}\frac{\partial U_i}{\partial x_j}-e_{ij}\right)$$

運動量方程式変換

$$\frac{\partial U_i}{\partial t}+\frac{\partial}{\partial \xi}\left\{\left(\boldsymbol{U}U_i+\frac{P}{\rho}\xi_{xi}-l_1U_{i\xi}-l_2U_{i\eta}-l_3U_{i\zeta}+\xi_x e_{i3}+\xi_y e_{i2}+\xi_z e_{i1}\right)\right\}$$

$$\frac{\partial}{\partial \eta}\left\{\left(\boldsymbol{V}U_i+\frac{P}{\rho}\xi_{xi}-l_2U_{i\xi}-l_4U_{i\eta}-l_5U_{i\zeta}+\eta_x e_{i3}+\eta_y e_{i2}+\eta_z e_{i1}\right)\right\}$$

$$\frac{\partial}{\partial \zeta}\left\{\left(\boldsymbol{W}U_i+\frac{P}{\rho}\xi_{xi}-l_3U_{i\xi}-l_5U_{i\eta}-l_6U_{i\zeta}+\zeta_x e_{i3}+\zeta_y e_{i2}+\zeta_z e_{i1}\right)\right\}=0$$

- 計量テンソル

$$l_1=d_{i1}\xi^2{}_{x1}+d_{i2}\xi^2{}_{x2}+d_{i3}\xi^2{}_{x3}$$

$$l_2=d_{i1}\xi_{x1}\eta_{x1}+d_{i2}\xi_{x2}\eta_{x2}+d_{i3}\xi_{x3}\eta_{x3}$$

$$l_3=d_{i1}\xi_{x1}\zeta_{x1}+d_{i2}\xi_{x2}\zeta_{x2}+d_{i3}\xi_{x3}\zeta_{x3}$$

$$l_4=d_{i1}\eta^2{}_{x1}+d_{i2}\eta^2{}_{x2}+d_{i3}\eta^2{}_{x3}$$

$$l_5=d_{i1}\eta_{x1}\zeta_{x1}+d_{i2}\eta_{x2}\zeta_{x2}+d_{i3}\eta_{x3}\zeta_{x3}$$

$$l_6=d_{i1}\zeta^2{}_{x1}+d_{i2}\zeta^2{}_{x2}+d_{i3}\zeta^2{}_{x3}$$

- 反変速度

$$\boldsymbol{U}=U_1\frac{\partial \xi}{\partial x_1}+U_2\frac{\partial \xi}{\partial x_2}+U_3\frac{\partial \xi}{\partial x_3}$$

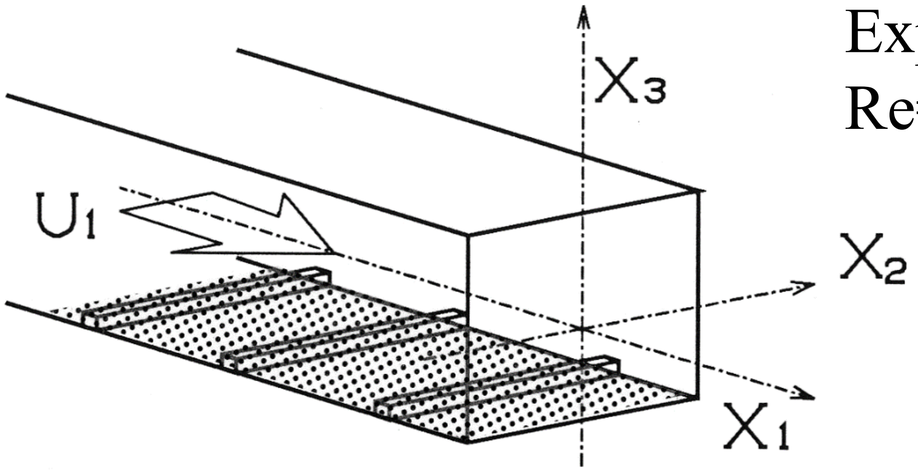
$$\boldsymbol{V}=U_1\frac{\partial \eta}{\partial x_1}+U_2\frac{\partial \eta}{\partial x_2}+U_3\frac{\partial \eta}{\partial x_3}$$

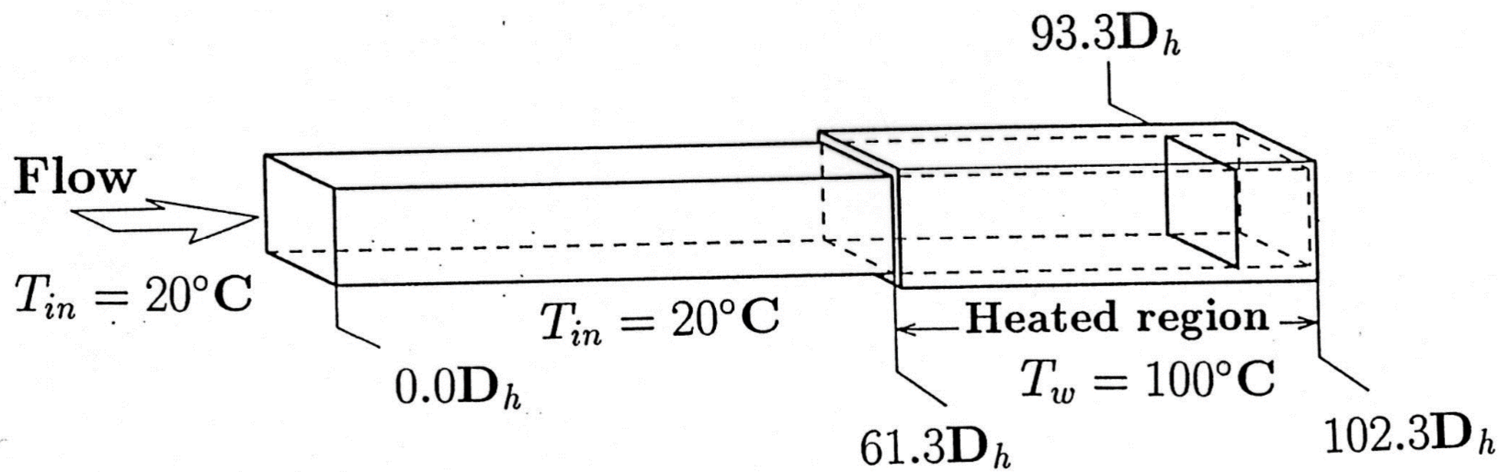
$$\boldsymbol{W}=U_1\frac{\partial \zeta}{\partial x_1}+U_2\frac{\partial \zeta}{\partial x_2}+U_3\frac{\partial \zeta}{\partial x_3}$$

$$\xi_{xi}=\partial\xi/\partial x_i \quad \eta_{xi}=\partial\eta/\partial x_i \quad \zeta_{xi}=\partial\zeta/\partial x_i$$

粗面壁の乱流熱伝達現象解析

Exp. by Fujita-Hirota(1992,1999)
Re=65,000





乱流の特徴的現象(第2種二次流れ)

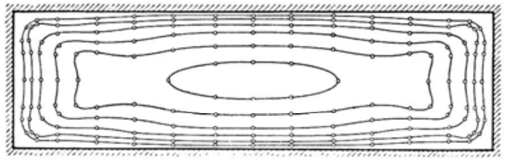


Fig. 20.13. Curves of constant velocity for a pipe of rectangular cross-section, after Nikuradse [43]

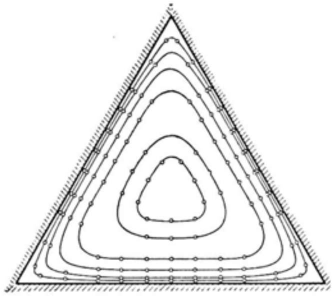


Fig. 20.14. Curves of constant ve locity for a pipe of equilateral tri angular cross-section, after Nikurads [44]

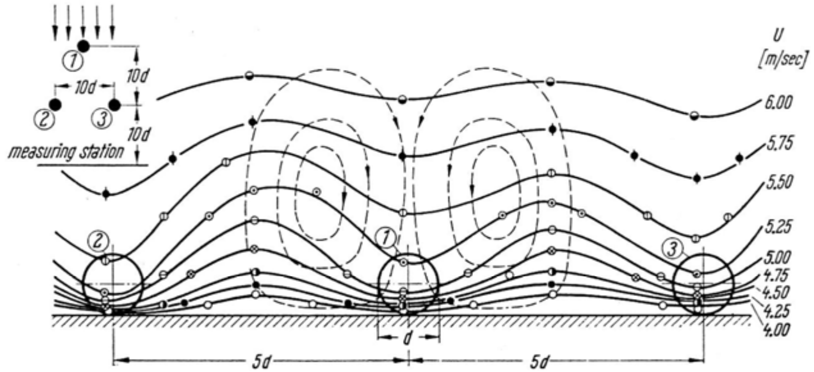


Fig. 21.13. Curves of constant velocity in the flow field behind a row of spheres (full lines), as measured by H. Schlichting [45], and accompanying it the secondary flow (broken lines) in the boundary layer behind sphere (1), as calculated by F. Schultz-Grunow [55a]. In the neighbourhood of the wall, the velocity behind the spheres is larger than that in the gaps. The spheres produce a "negative wake effect" which is explained by the existence of secondary flow
Diameter of spheres $d = 4$ mm

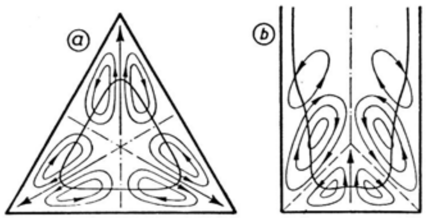


Fig. 20.15. Secondary flows in pipes of triangular and rectangular cross-section (schematic)

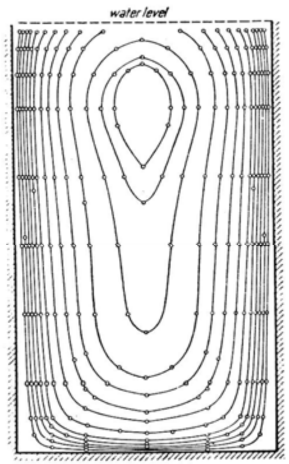
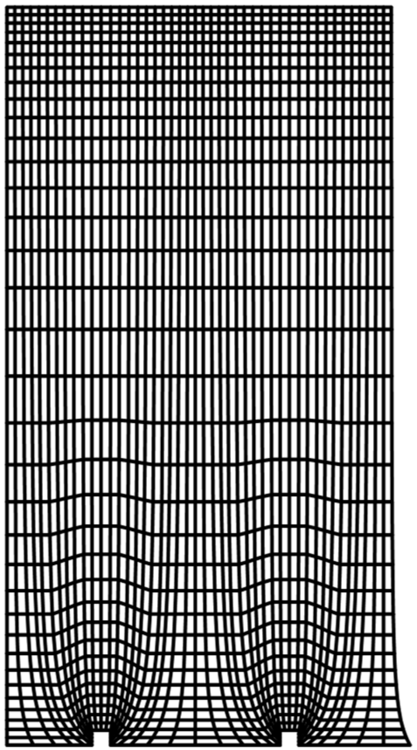


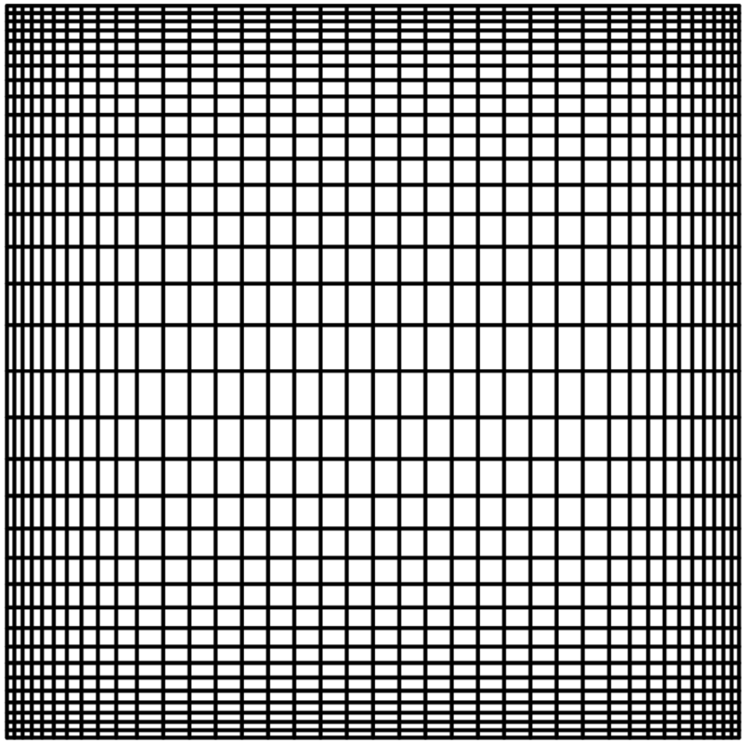
Fig. 20.16. Curves of constant velocity for a rectangular open channel, after Nikuradse [43]

Boundary Layer Theory
by Hermann Schlichting

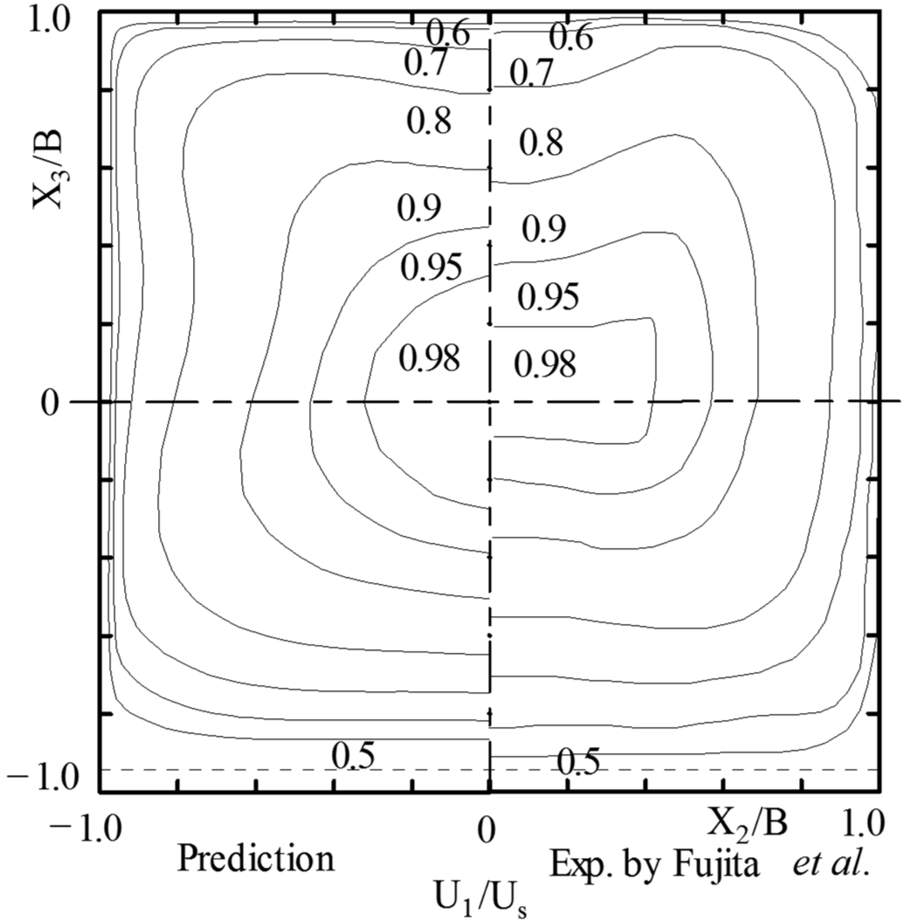
計算格子

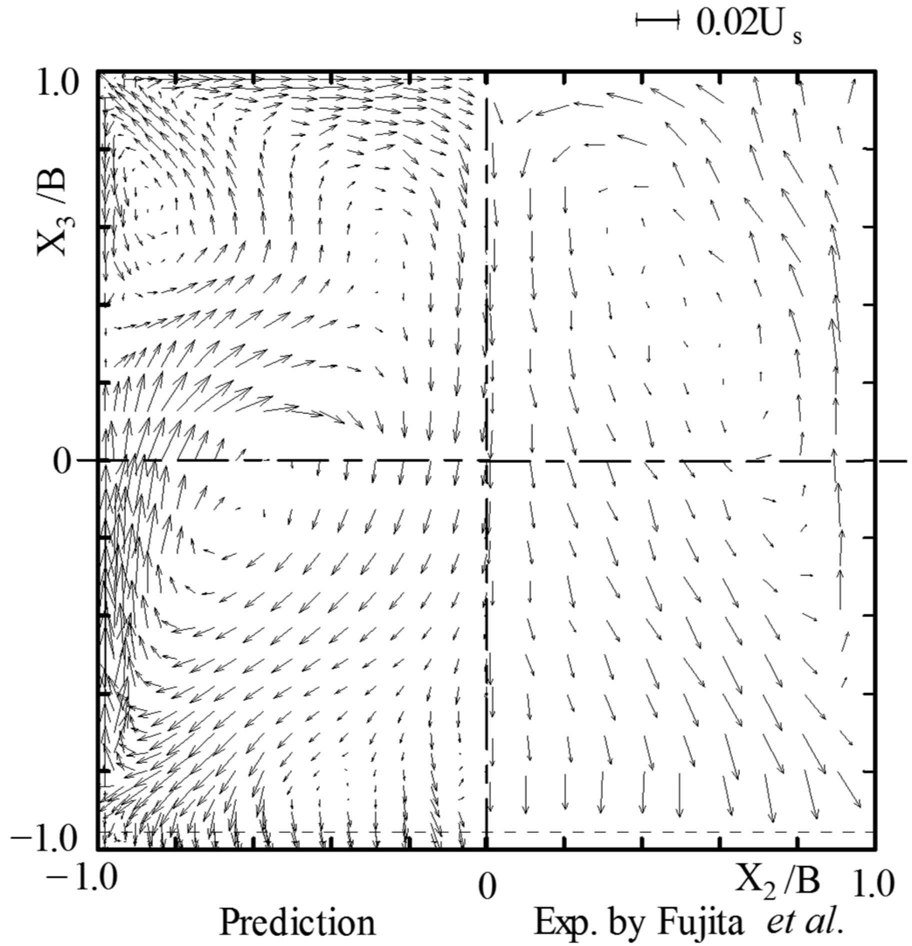
- 周期境界条件を課して解析
- 計算格子数：39x35x46=62,796



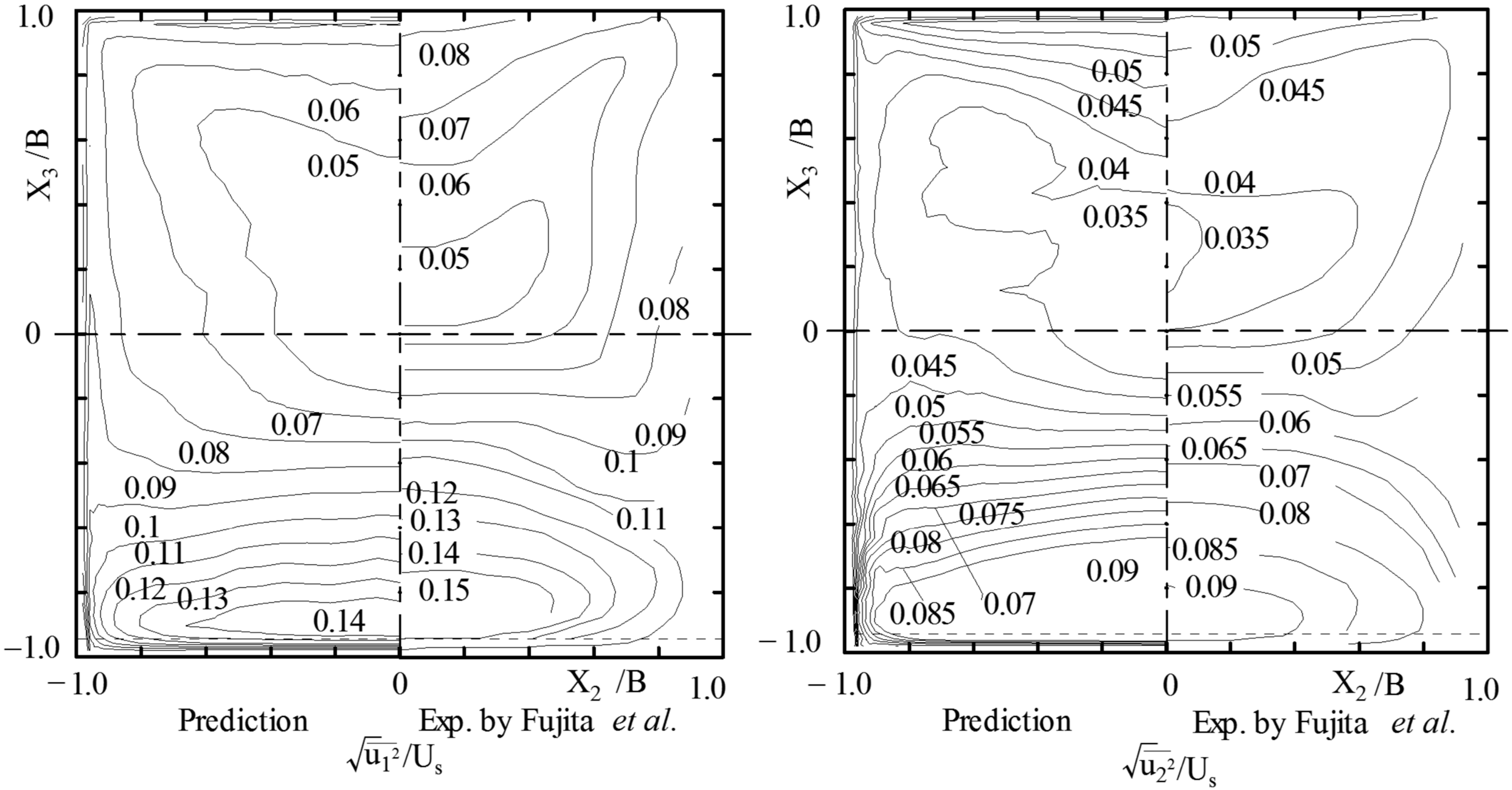


主流方向速度，第2種二次流れ比較

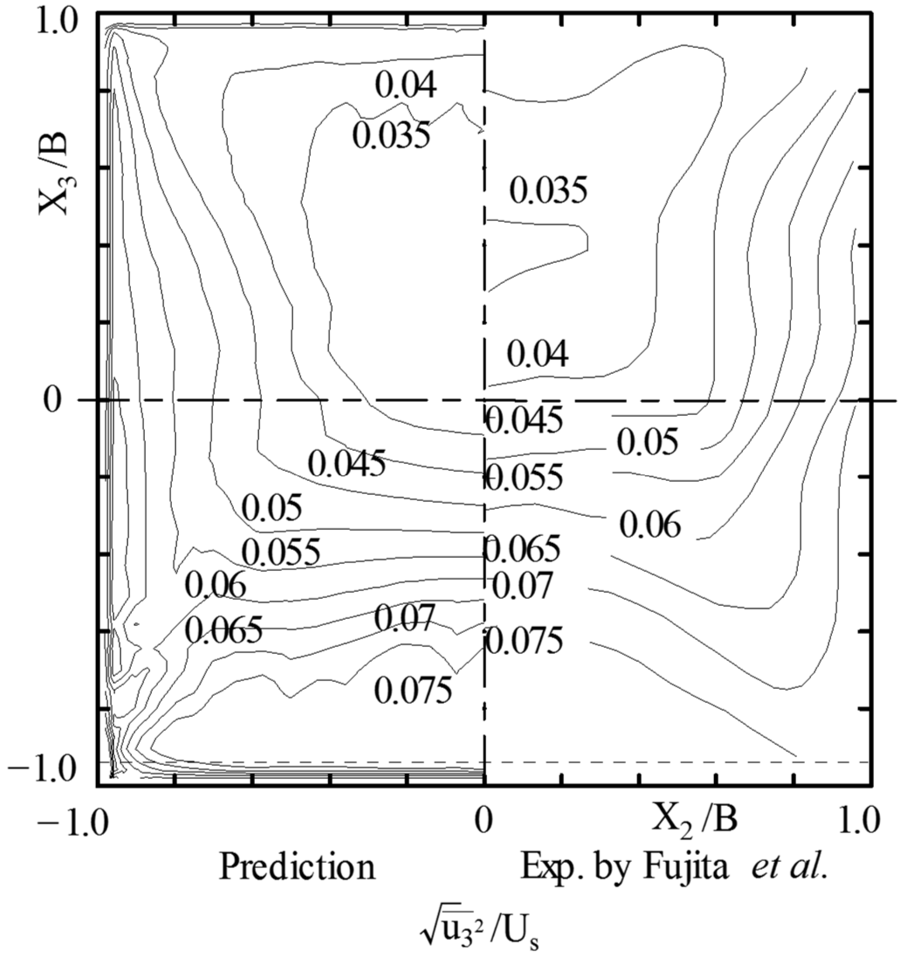




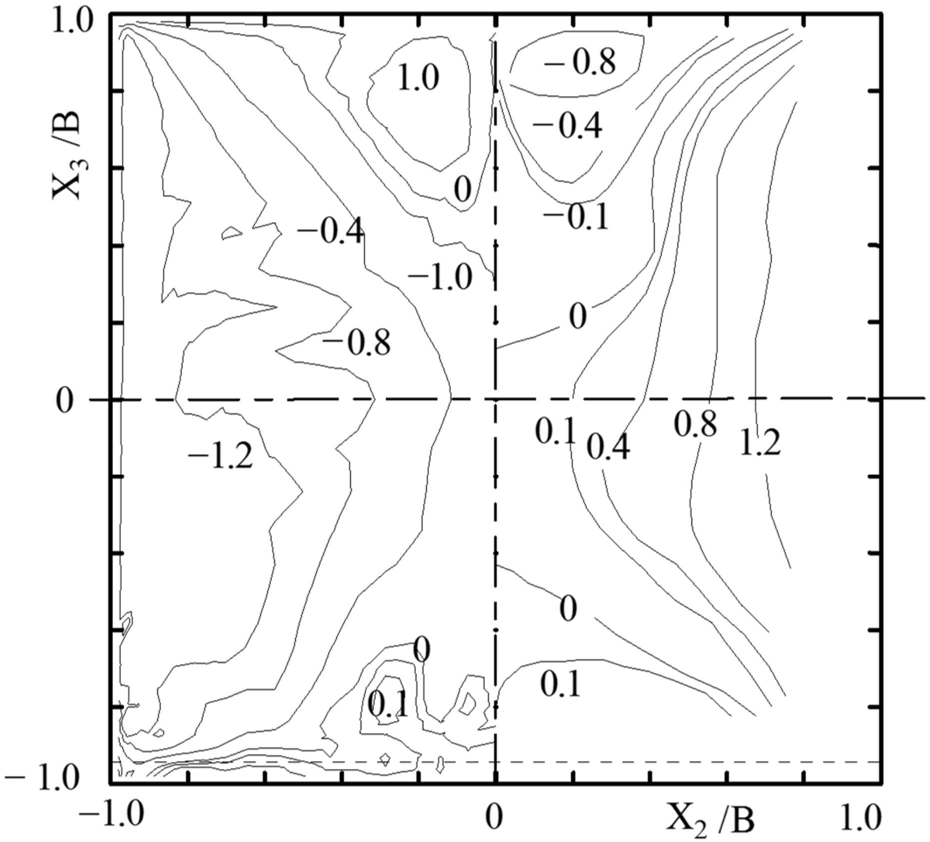
垂直応力 $\sqrt{\overline{u_1^{\,2}}}$, $\sqrt{\overline{u_2^{\,2}}}$ 比較



垂直応力 $\sqrt{\overline{u_3^2}}$ 比較

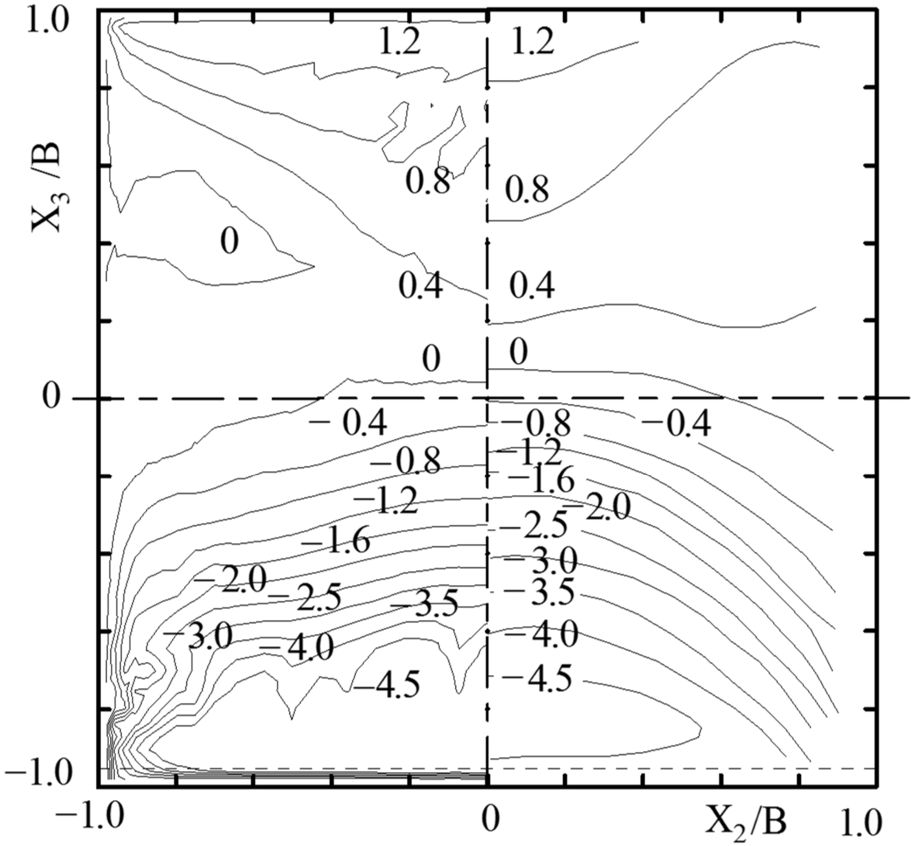


せん断応力 $\overline{u_1u_2}$, $\overline{u_1u_3}$ 比較



Prediction　　Exp. by Fujita *et al.*

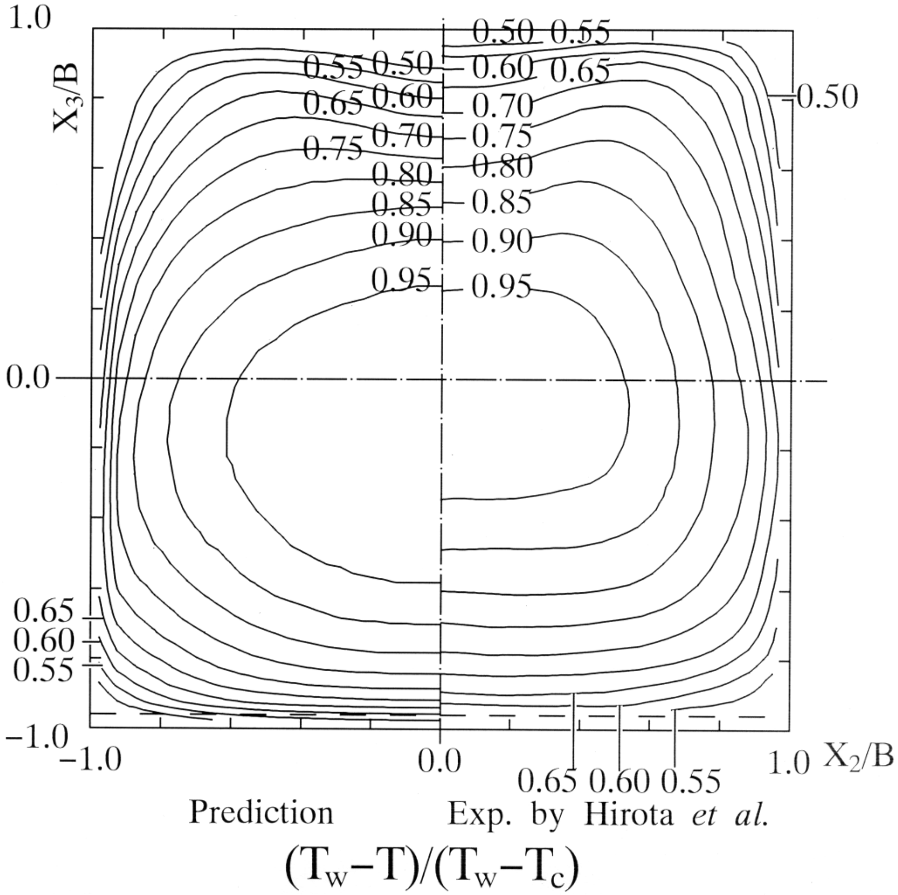
$(\overline{u_1u_2}/U_s^2)\times10^3$



Prediction　　Exp. by Fujita *et al.*

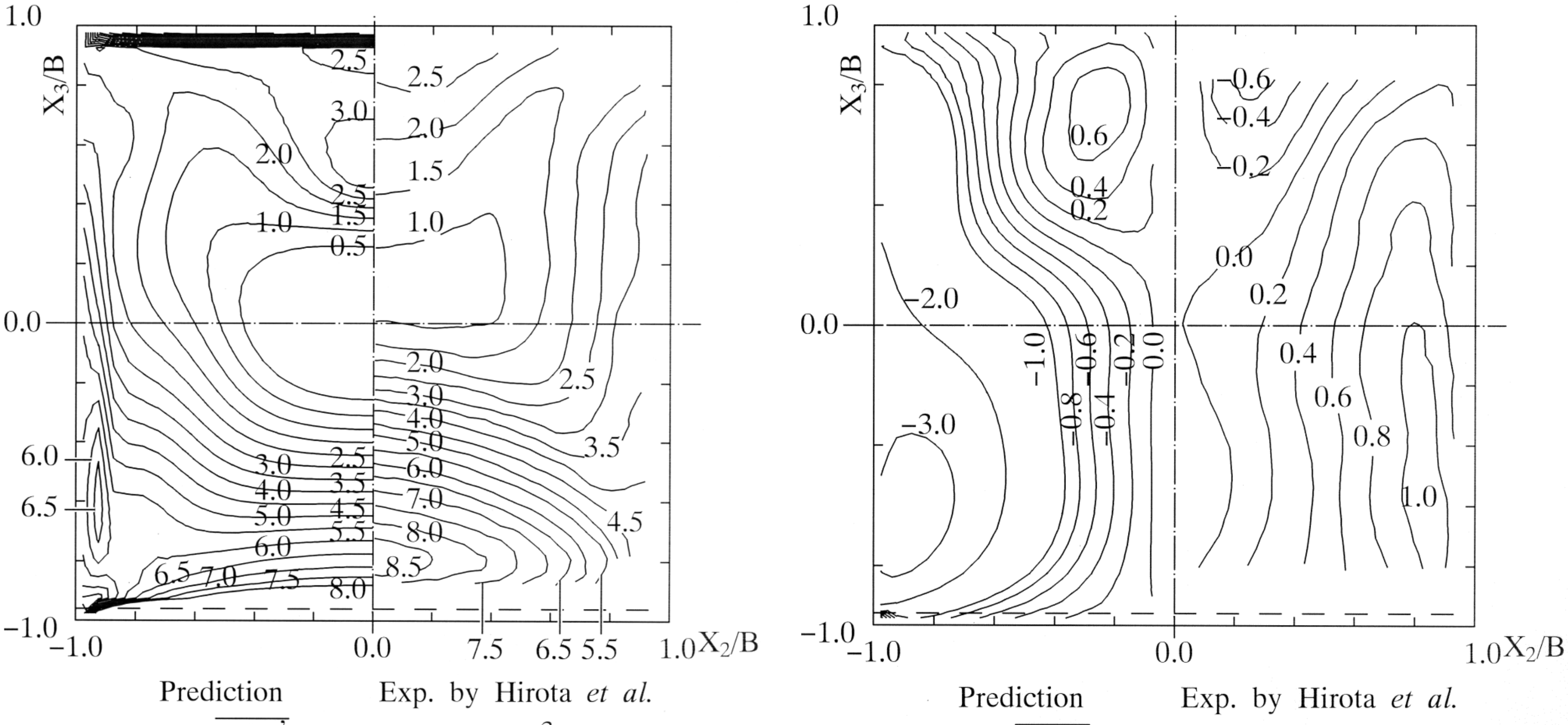
$(\overline{u_1u_3}/U_s^2)\times10^3$

平均温度分布比較



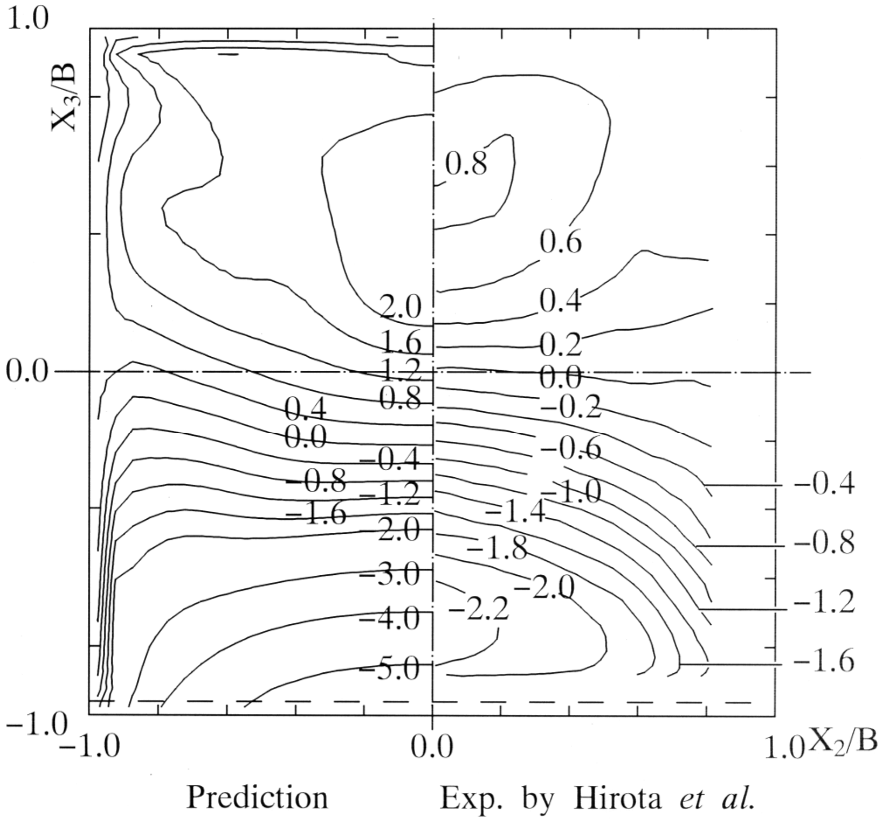
1.0
0.0
−1.0
X_3/B
0.55
0.50
0.65
0.60
0.75
0.70
0.80
0.85
0.90
0.95
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
0.50
0.65
0.60
0.55
−1.0
0.0
1.0
X_2/B
0.65
0.60
0.55
Prediction
Exp. by Hirota et al.
$(T_w-T)/(T_w-T_c)$

乱流熱流束 $-\overline{u_1T'}$, $-\overline{u_2T'}$ 比較



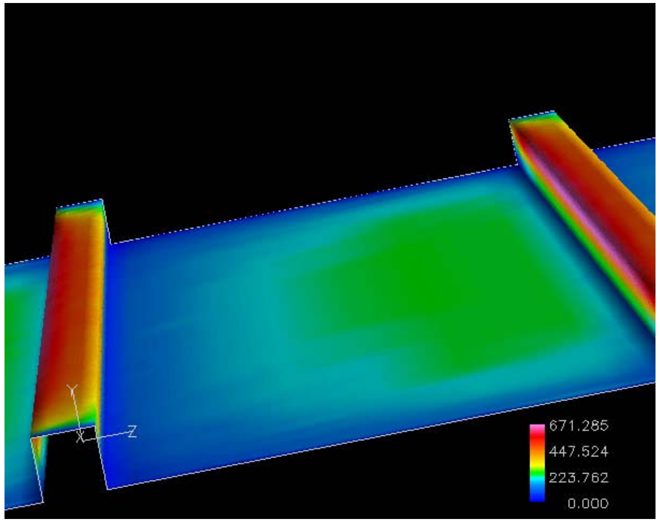
$(-\overline{u_1T'}/U_c(T_w-T_c))\times10^3$ 　 $(-\overline{u_2T'}/U_c(T_w-T_c))\times10^3$

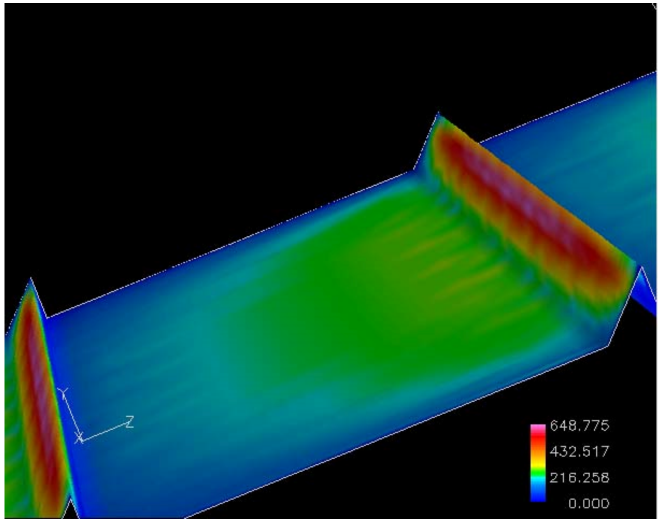
乱流熱流束 $-\overline{u_3 T'}$ 比較

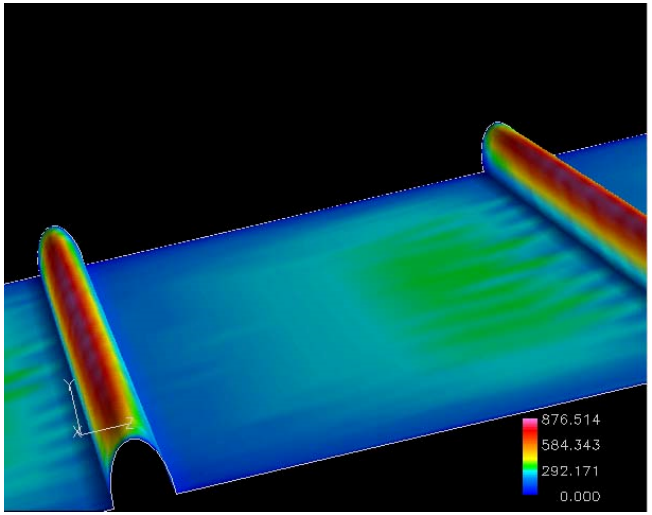


$(-\overline{u_3 T'}/U_c(T_w - T_c)) \times 10^3$

突起形状によるヌセルト数分布







Washington大学滞在

- 文部省在外研究員制度：
- Host 教授： Prof. Fred Gessner
- 滞在目的：非等方性乱流モデル研究
  乱流熱流束モデル研究
  粗面壁乱流熱伝達現象解析
- 滞在期間：2000.6.1～2001.3.31



Thrust vectoring nozzles内の乱流解析

Exp. by Davis and Gessner (1992)
Re=390,000

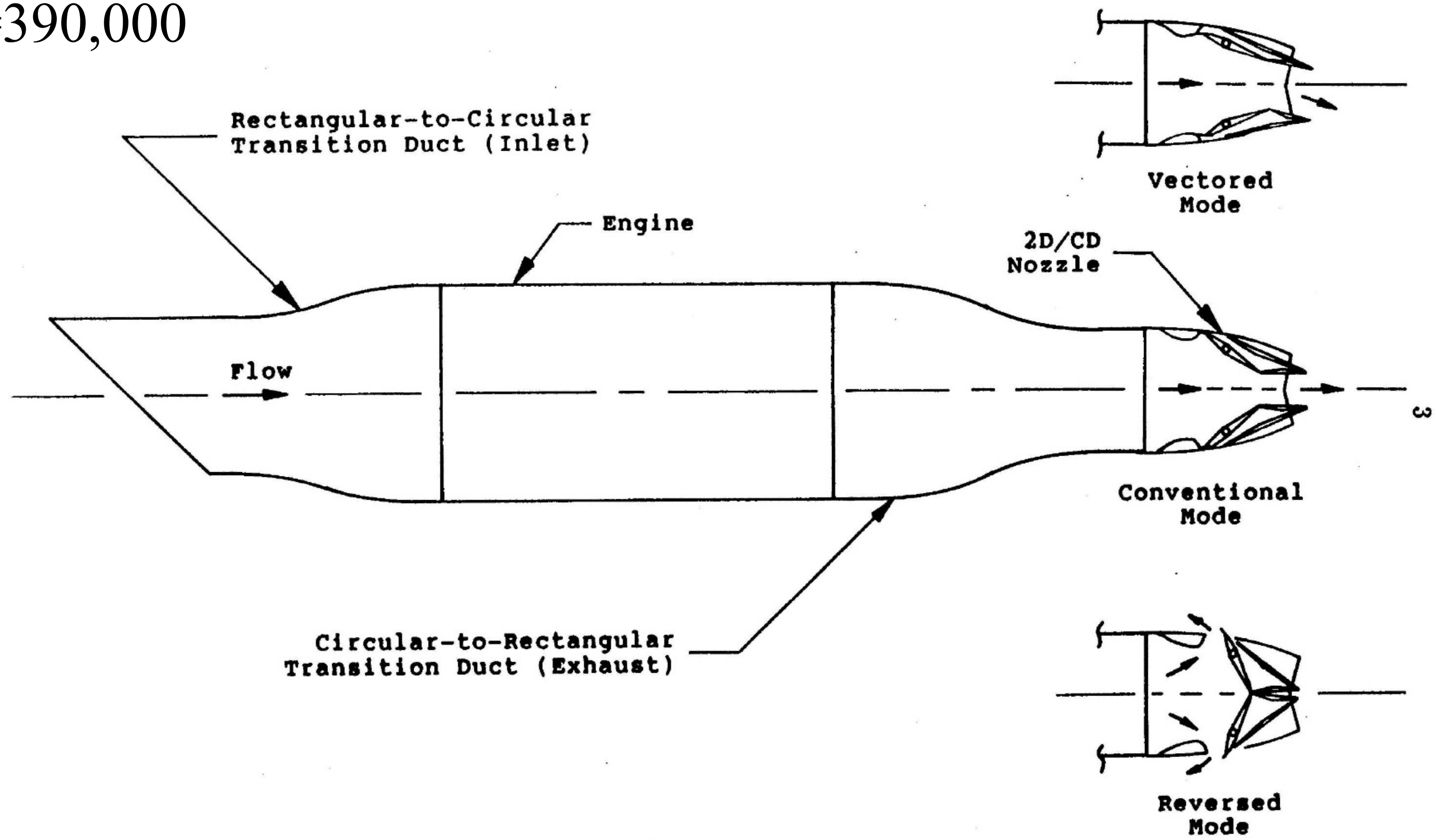
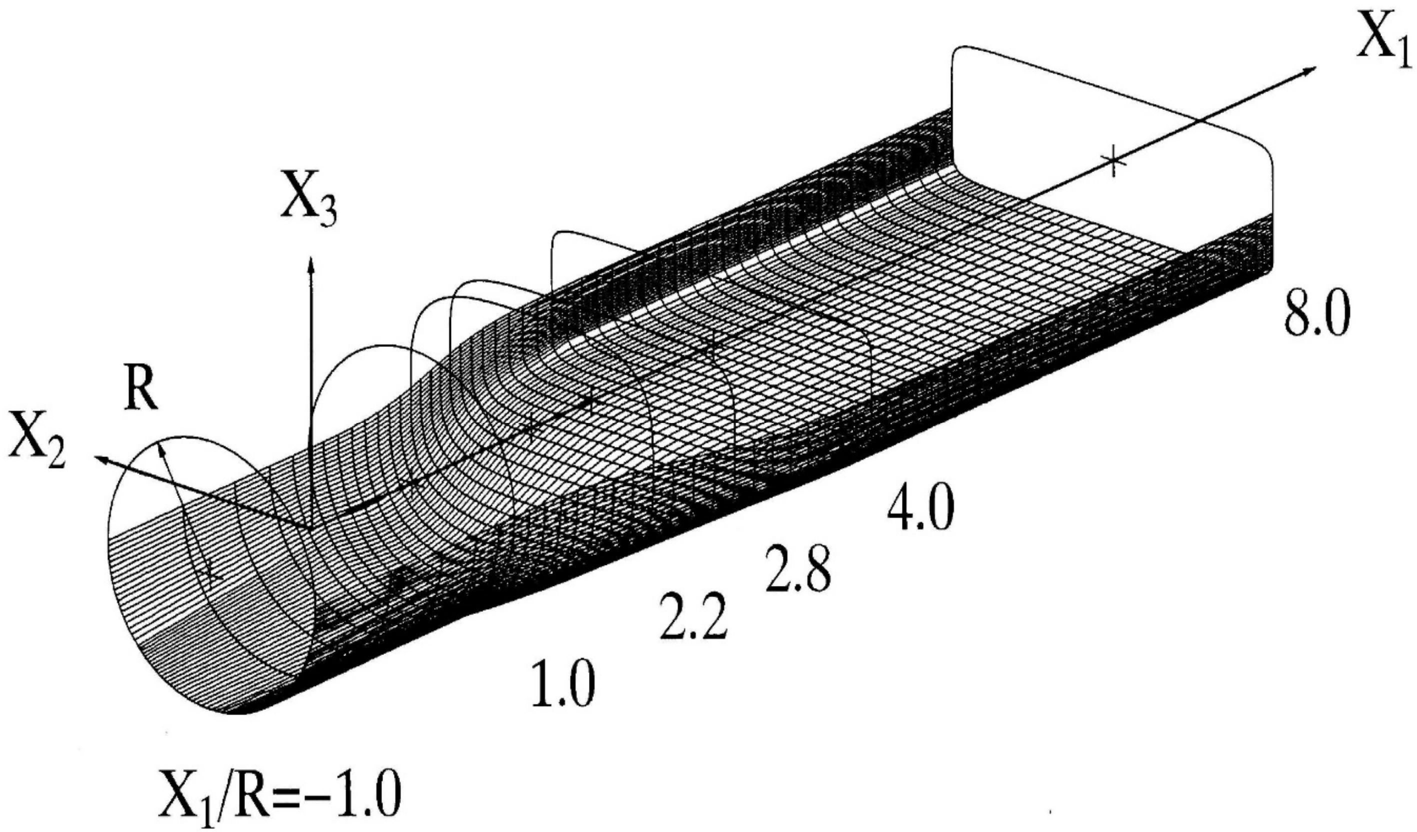


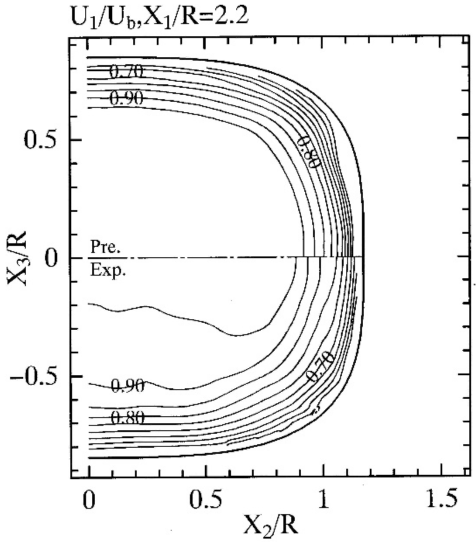
Fig. 1.1. Schematic of typical propulsion system.

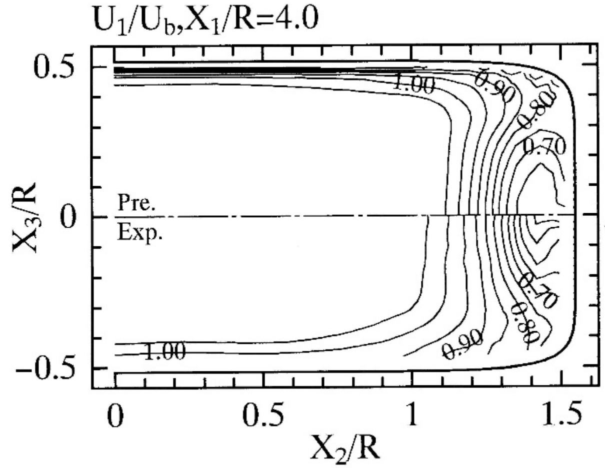
計算格子

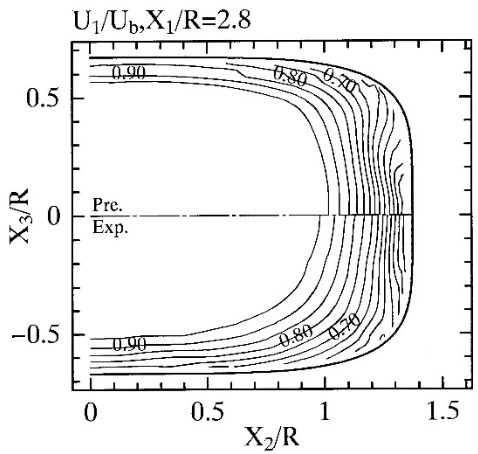


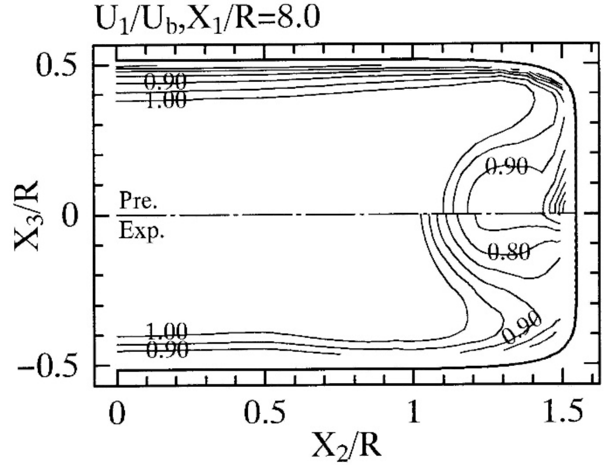
- 計算格子数：120x25x20=65,000 （1/4断面）

主流方向速度の比較

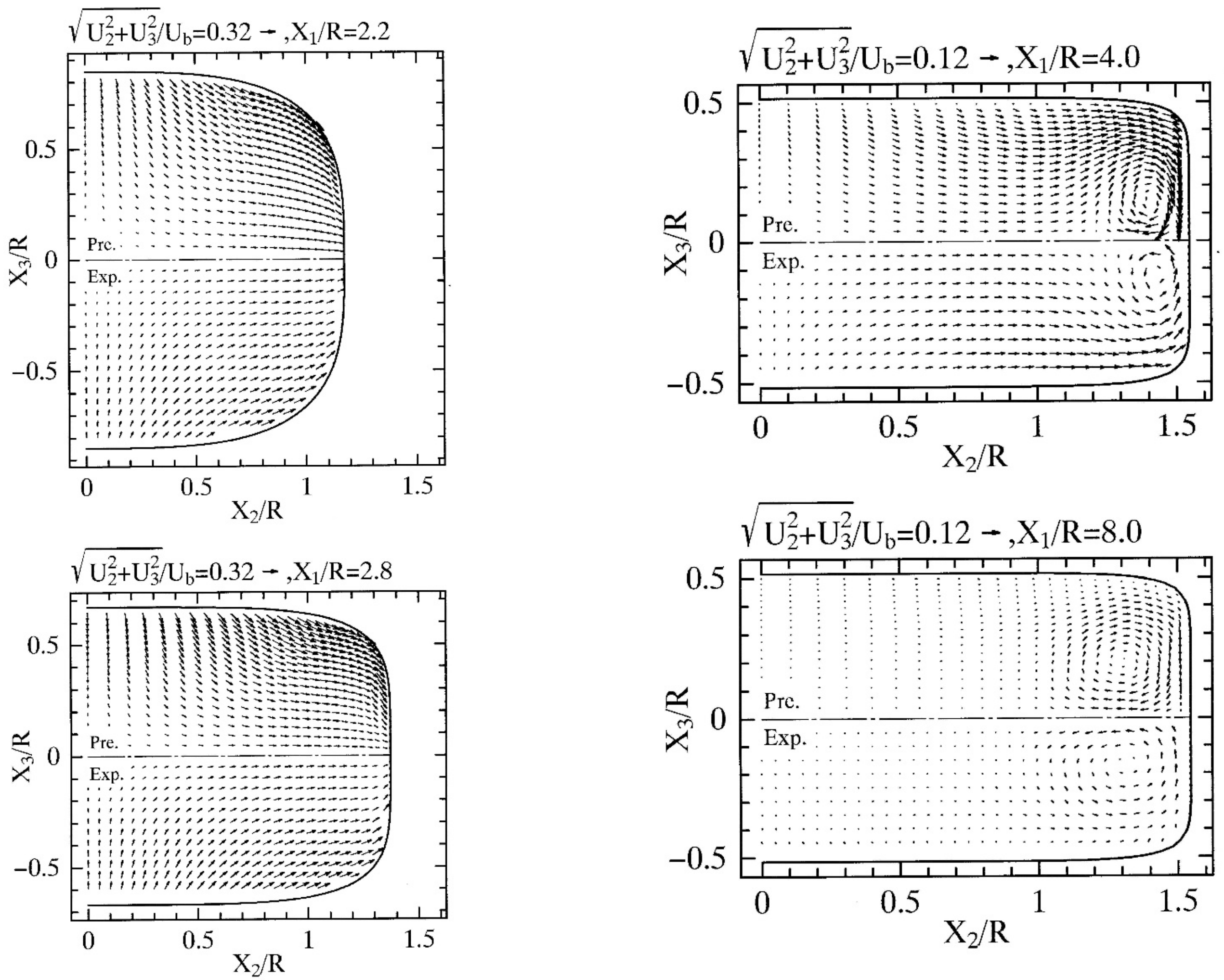

U1/Ub,X1/R=2.2
0.70
0.90
0.80
Pre.
Exp.
0.90
0.80
0.70
0.5
0
−0.5
X3/R
0
0.5
1
1.5
X2/R


U1/Ub,X1/R=4.0
1.00
0.90
0.80
0.70
Pre.
Exp.
0.70
0.80
0.90
1.00
0.5
0
−0.5
X3/R
0
0.5
1
1.5
X2/R

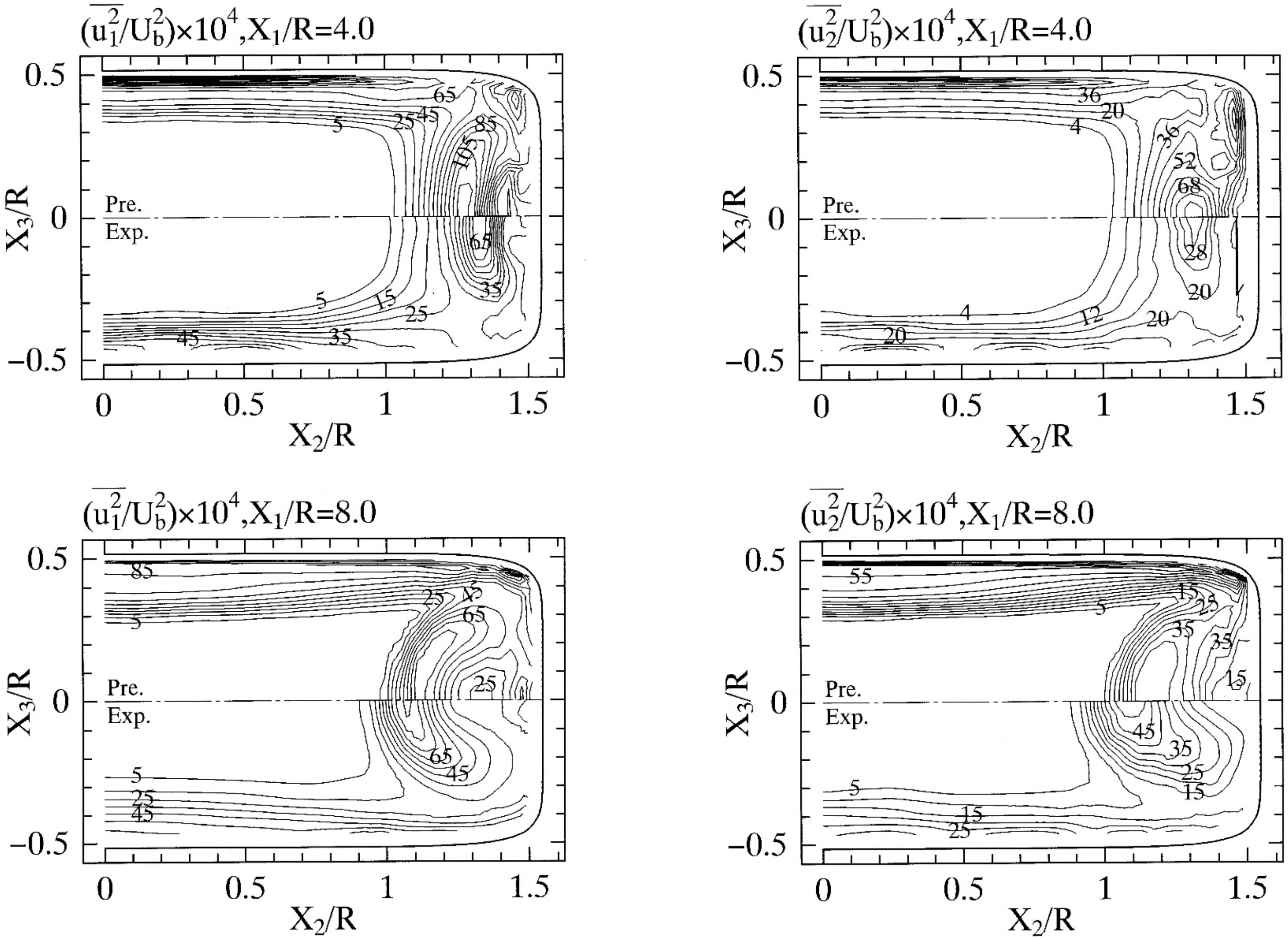

U1/Ub,X1/R=2.8
0.90
0.80
0.70
Pre.
Exp.
0.90
0.80
0.70
0.5
0
−0.5
X3/R
0
0.5
1
1.5
X2/R


U1/Ub,X1/R=8.0
0.90
1.00
0.90
Pre.
Exp.
0.80
0.90
1.00
0.90
0.5
0
−0.5
X3/R
0
0.5
1
1.5
X2/R

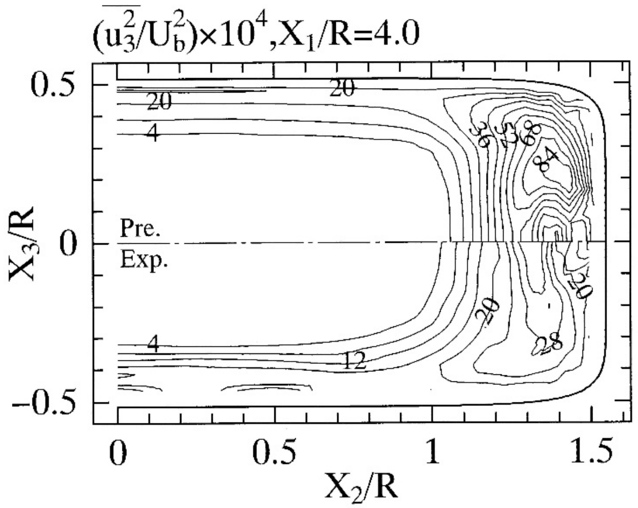
二次流れの比較(第1種二次流れ)

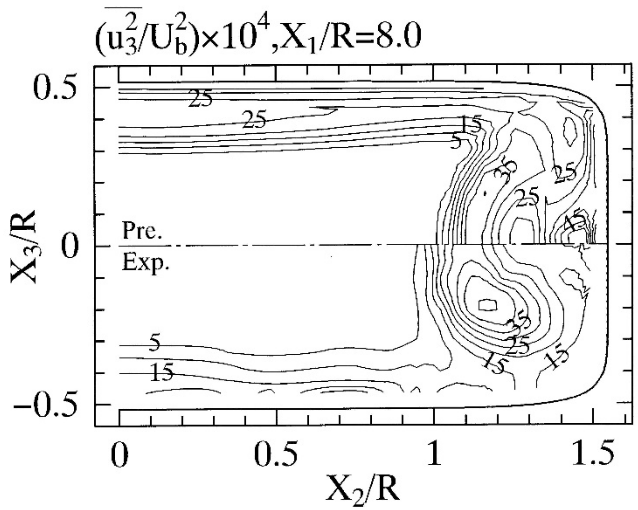


垂直応力 $\sqrt{\overline{u_1}^2}$, $\sqrt{\overline{u_2}^2}$ 比較

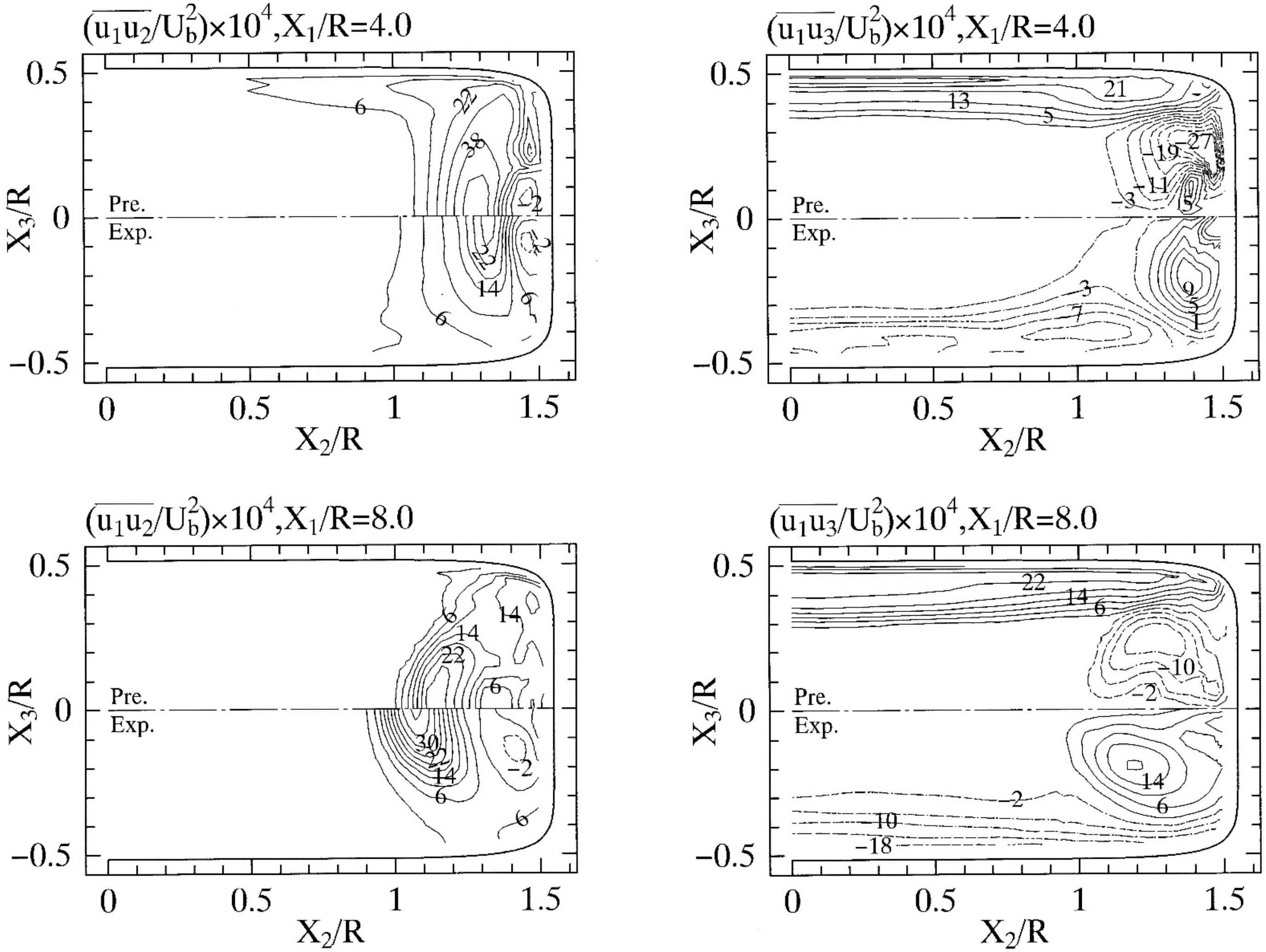


垂直応力 $\sqrt{\overline{u_3^{\,2}}}$ 比較





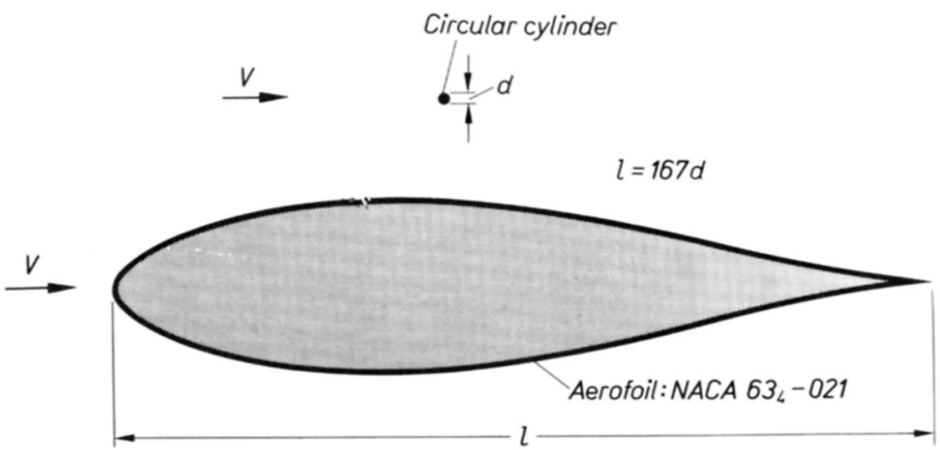
せん断応力 $\overline{u_1u_2}$，$\overline{u_1u_3}$ 比較



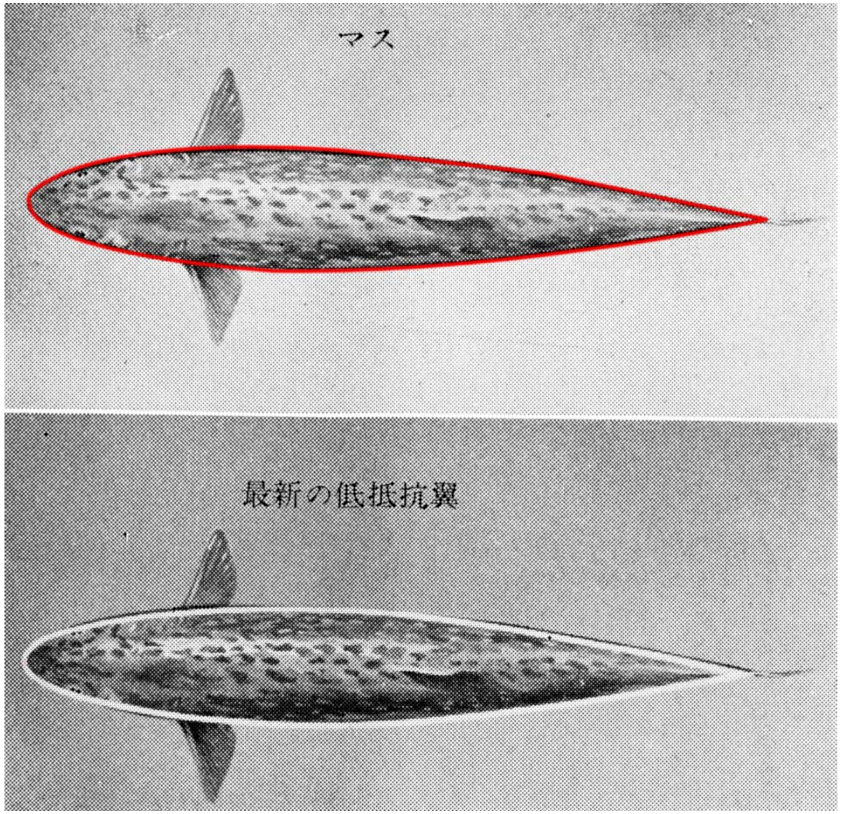
F-22, Raptor

Lockheed-Martin/ Boeing
Stealth, STOL, Super cruising
2005年運用開始

魚の泳法メカニズムと熱流動解析

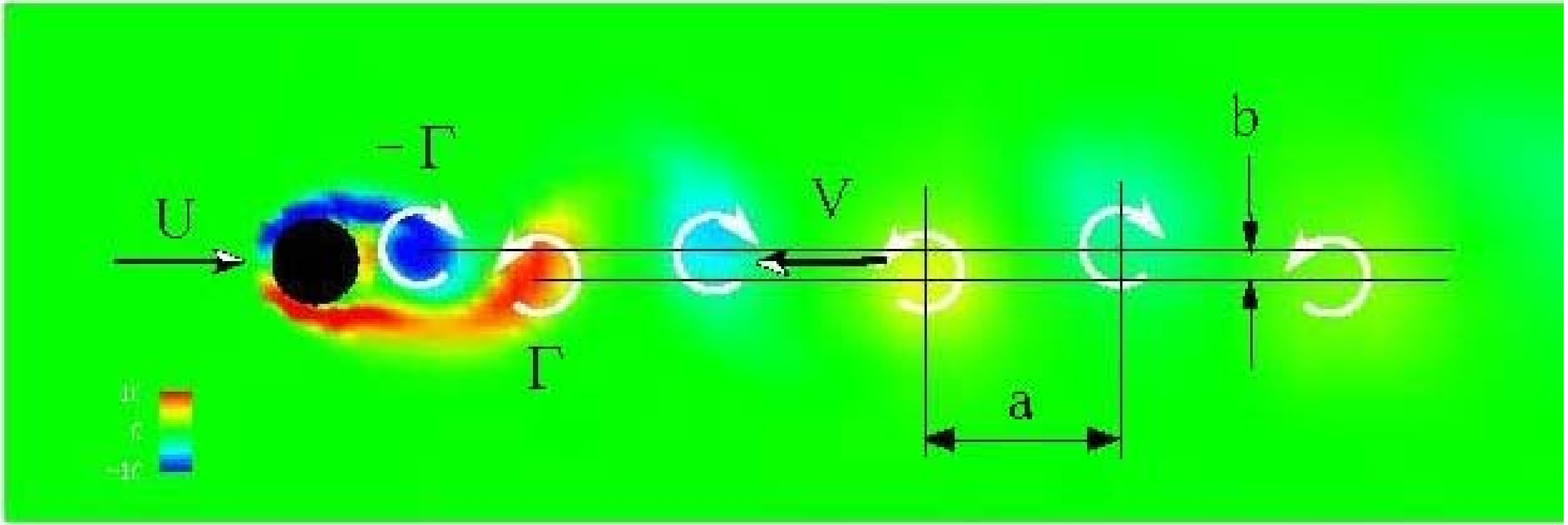


Boundary Layer Theory
by Hermann Schlichting



Shape and Flow
by Ascher H. Shapiro

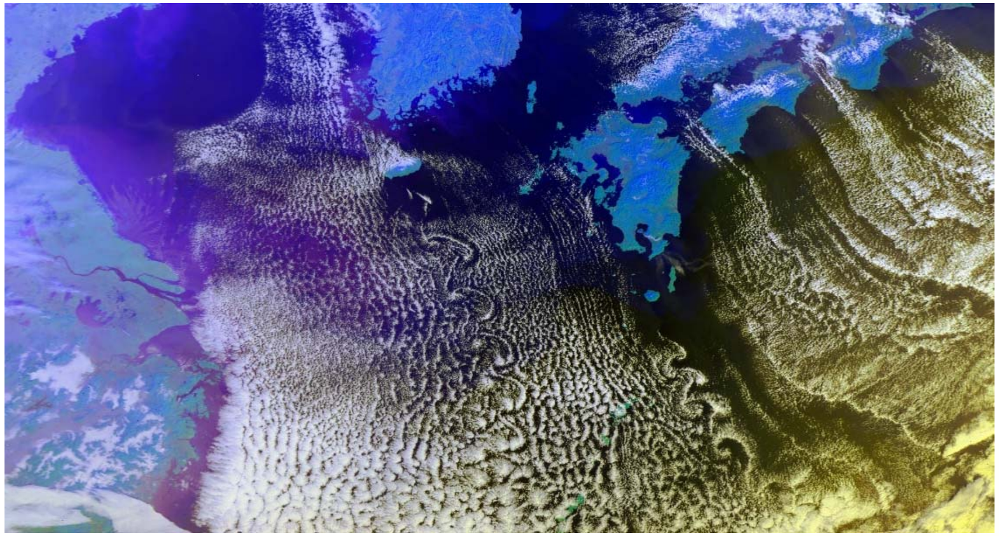
カルマン渦列



$$D = \frac{\rho \Gamma b}{a}(U - V) + \frac{\rho \Gamma^2}{2\pi a} \quad , \qquad \Gamma = \oint \vec{v} d\vec{s}$$

- D:抵抗，Γ:循環（反時計方向を正），ρ :密度
- V:静止流体に対する渦列の移動速度

自然界のカルマン渦列



済州島，屋久島のカルマン渦

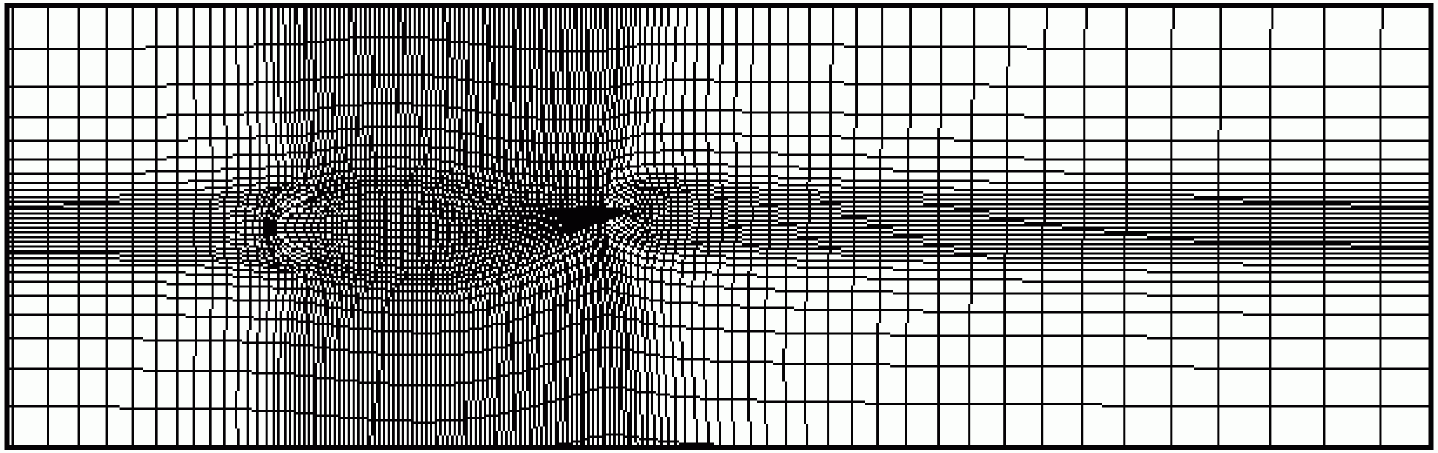
Tacoma narrows bridgeの崩壊

解析手法

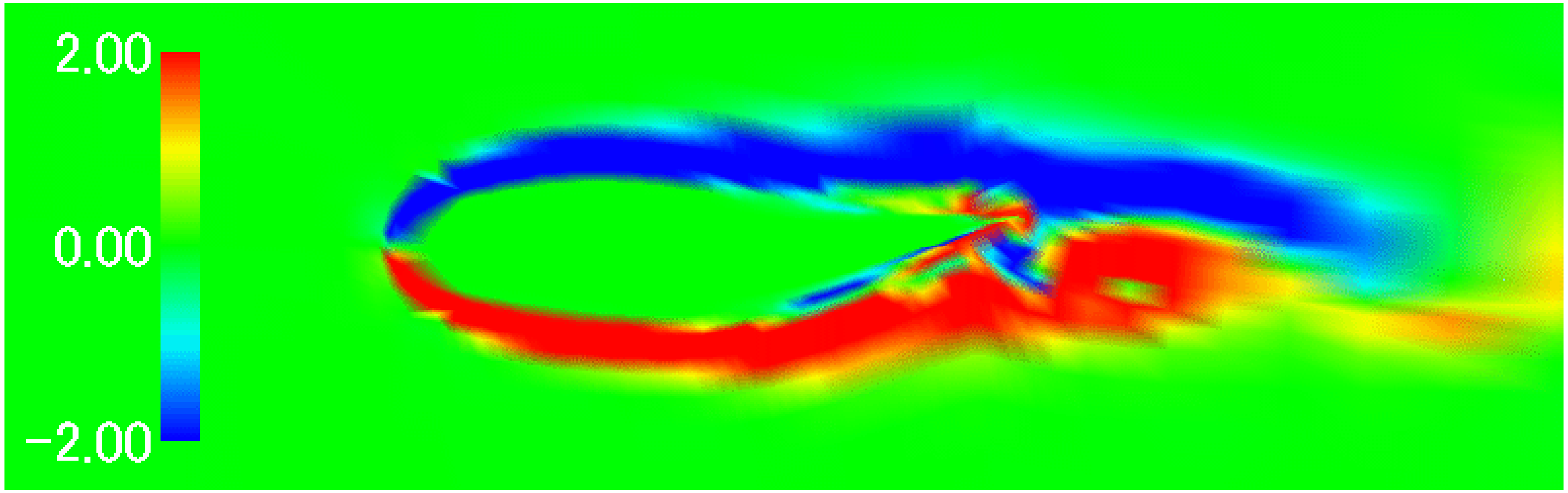
- 魚の動き

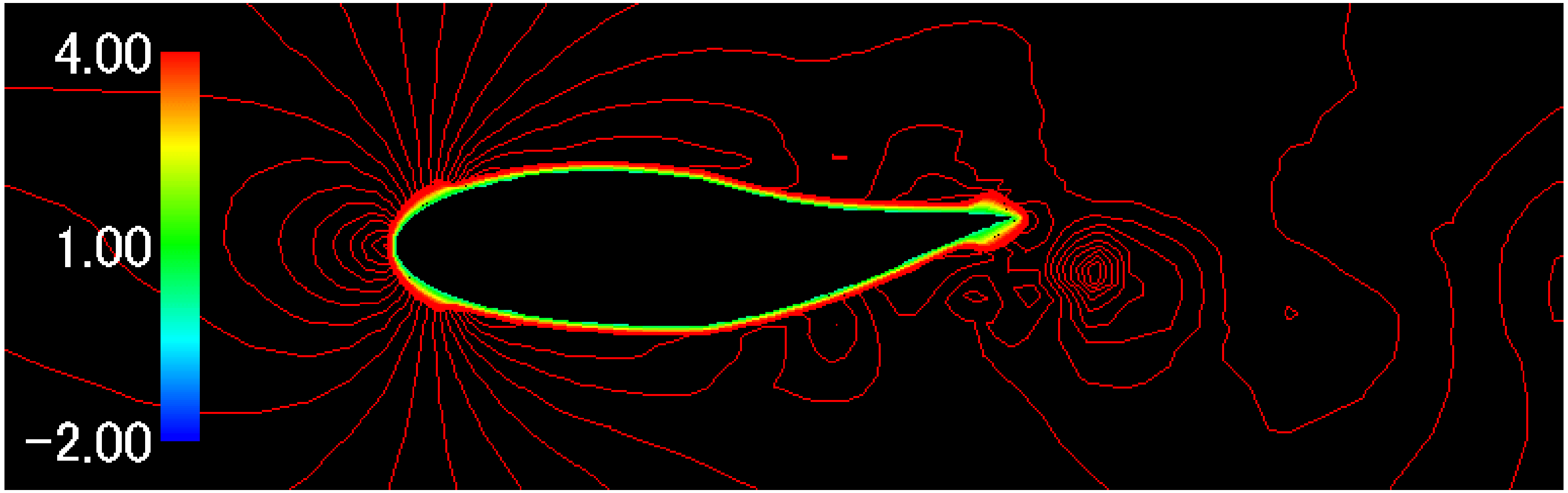
$$h = 0.04744 X^2 \cos\left[2\pi\left(\frac{t}{T} - X\right)\right]$$

- H：中心線からの変位，X：魚の先端からの距離
- T：1周期時間(2400x2.5x10^4)，t ：1ステップ計算時間
- 計算レイノルズ数　Re＝500

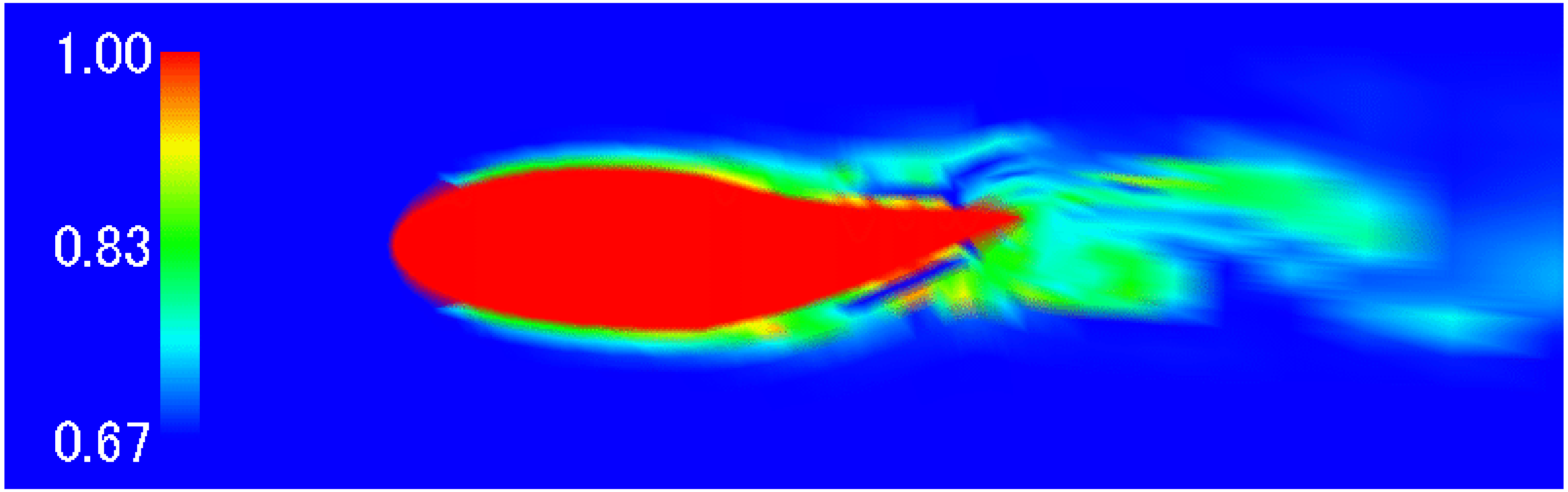


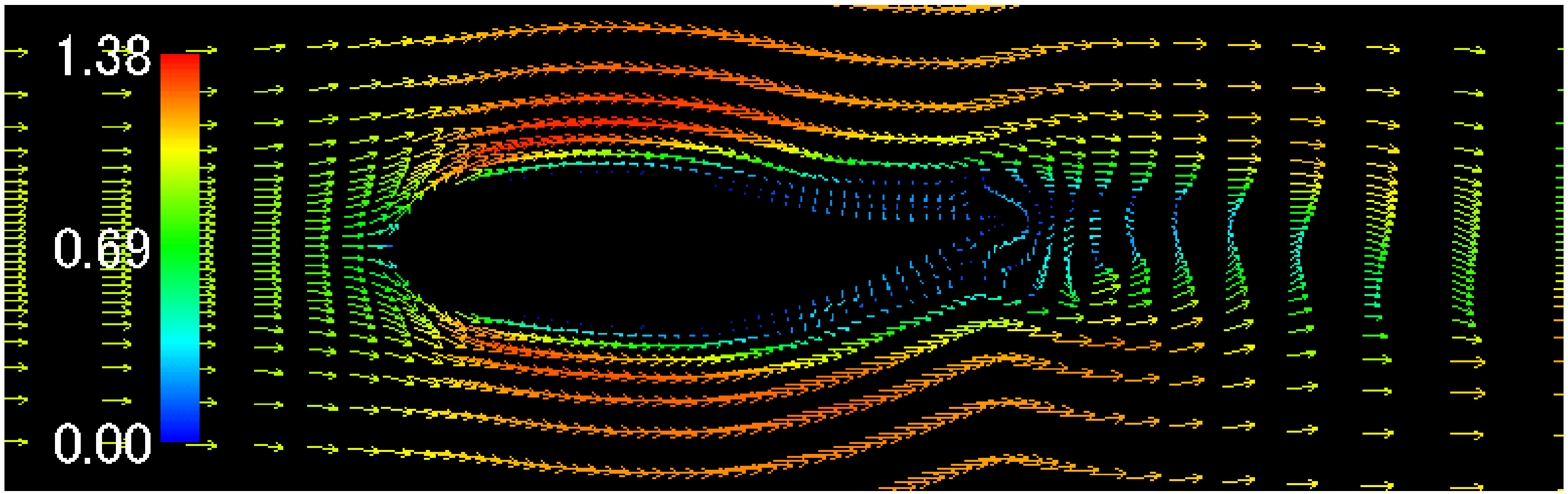
魚の泳法(渦度分布, 圧力分布)





魚の熱移動





ご清聴
ありがとうございました。

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